Global Linear Instability on Laminar Separation Bubbles – revisited: U-separation and the Birth of Stall Cells on Airfoils

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Abstract

The topologies of the composite flowfields, reconstructed by linear superposition of two-dimensional separated basic flows and their respective leading three-dimensional global eigenmodes, have been studied in two configurations of both academic and industrial interest: a transitional laminar separation bubble in two-dimensional adverse-pressure-gradient boundary layer flow and a stalled NACA0015 airfoil. The amplitudes considered in both linear superpositions are small enough for the linearization assumption to be valid. On the flat plate, where both the Tollmien-Schlichting and the global mode instabilities are operative, very good agreement between the results of the local and global analyses has been obtained as far as the TS instability mechanism is concerned. In the same configuration, the origins of the phenomenon of *U-separation* have been attributed to linear amplification of the global mode. On the other hand, on both the flat plate and the stalled airfoil, it is shown that amplification of the respective leading stationary global flow eigenmode leads to the degenerate basic flow topology being replaced by fully three-dimensional patterns, which are strongly reminiscent of the characteristic *Stall Cells*, observed experimentally on airfoils, concurrently with the onset of stall in both laminar and turbulent flow.

1. Introduction

Two phenomena associated with separated flow have long been known and have been described in the literature either experimentally or from a qualitative point of view. The first is *U-separation* [6, 3], one of the possible forms in which a separated flow may be classified using critical point theory; the second is the self-organization of separated flow on the suction side of a plane rectangular wing and/or airfoil just beyond stall into three-dimensional spanwise-periodic structures known as *Stall Cells.* Most of the phenomenological knowledge regarding stall cells has been obtained experimentally using oil and smoke visualization techniques on finite-wing models [2, 16]. Their emergence was shown not to be a tip effect [12, 5], but the result of a periodic spanwise breakdown of the separated flow region. The physical origins of both U-separation and the three-dimensional breakdown of the two-dimensional baseline flow are presently unknown.

In the continued quest for a physical understanding of these phenomena, which can lead to theoretically-founded flow control methodologies, critical-point theory has been employed to the study of separated flow topology by several investigators. This theory emerged in the context of fluid flow analysis in the early 80's of the last century [4, 6, 8, 3] and rapidly became a powerful tool in describing flow patterns in both laminar and turbulent flow. Critical point theory asserts that two-dimensional flow topologies are defined as degeneracies and any three-dimensional disturbance will lead to a new three-dimensional flowfield topology, regardless of the disturbance amplitude. To-date, different, and on occasion contradictory topological descriptions (i.e. characterization of the critical points and connecting streamlines) of laminar separation may be found in the literature, a fact which is hardly surprising given the richness of the different geometries on and flow conditions under which the phenomenon of separation appears. However, consensus does exist in the description of stall cells on separated airfoils as symmetric, counter-rotating swirling structures.

Flow instability is known to play a decisive role in configurations where laminar separation exists. Concurrently with the emergence of global instability ideas, either in its interpretation as absolute instability of weakly non-parallel flows or as linear modal instability of strongly non-parallel flows [13], analyses of instability of laminar separation bubbles on a flat-plate boundary layer have been performed. Absolute/convective instability analysis of detached boundary layers has shown the existence of a strong two-dimensional instability of the Kelvin-Helmholtz class; the same methodology, applicable to weakly-non-parallel flows of the boundary-layer type, has hinted at the possibility of self-excitation of laminar separation bubbles embedded inside a boundary layer. On the other hand, the solution of the partial-derivative eigenvalue problem without resorting to the assumption of weak non-parallelism has provided unequivocal demonstrations of the potential of recirculating flows to self-excite an intrinsic instability mechanism [14], both in laminar separation bubbles in boundary-layer flow and in geometry-induced separation, such as the archetypal backward-facing step configuration. The key identification characteristics of this self-excited mechanism, namely its stationary and three-dimensional nature, differentiate it from the Tollmien-Schlichting / shear-layer instability. Hereafter, the self-excitation mechanism of instability is referred to as *the global mode of laminar separation*, in lieu of the necessity to relax the assumption of weak-nonparallelism of the basic state in order to be able to analyze instability of massively separated flow, such as that associated with stalled airfoils.

The common framework of global (also referred to as BiGlobal [14]) instability analysis, whereby a two-dimensional strongly nonparallel basic state is considered and small-amplitude perturbations are two-dimensional functions of the resolved spatial coordinates – the third spatial direction is treated as homogeneous – permits addressing instability of both boundary-layer flows and massive-, bluff-body-like separation in a self-consistent manner. The connection of BiGlobal theory and topological flow considerations is made on account of the assertion of critical-point theory that any two-dimensional description (here the so-called basic flow of global instability analysis) is defined as a degeneracy, only possible in strictly two-dimensional flow, which is to be replaced by a fully three-dimensional flow topology when an amplified (global) perturbation is present in the flow, regardless of the disturbance amplitude.

The present contribution thus seeks to compare the topological analyses of two separated flow configurations, in both of which amplified global modes exist, namely a flat-plate boundary-layer with an embedded laminar separation bubble and a NACA0015 airfoil at stalled conditions. The paper is organized as follows: Section 2 presents the linear BiGlobal instability analysis theory, the two basic flow configurations and some remarks on the theory of critical points. The results of the instability analyses, along with the topologies arising from the flow reconstructions are presented in Section 3. A discussion of the present findings as regards U-separation, stall cells and the qualitative differences between topologies in the two configurations studied is furnished in Section 4. Full theoretical details on each of the two configurations discussed herein are respectively provided in two accompanying publications [10, 11], to which the interested reader is also referred.

2. Theory

2.1 BiGlobal instability analysis

In the scope of a BiGlobal instability analysis [13] of the flows monitored herein, both considered incompressible and laminar, the three-dimensional unsteady flow field $\boldsymbol{q} = (u, v, w, p)^T$ is decomposed into a two-dimensional $\overline{\boldsymbol{q}}$ and a small-amplitude unsteady three-dimesional part, $\widetilde{\boldsymbol{q}}$, according to the Ansatz

$$\boldsymbol{q} = \overline{\boldsymbol{q}} + \varepsilon\, \widetilde{\boldsymbol{q}} + c.c. \tag{1}$$

The quantity $\overline{\boldsymbol{q}} = [\bar{u}(x,y,t), \bar{v}(x,y,t), \bar{w}(x,y,t), \bar{p}(x,y,t)]^T$ is the so-called basic flow, while x and y respectively denote the streamwise and wall-normal spatial directions. The basic flow here is taken to be steady and laminar, having two velocity components, i.e. no crossflow has been considered, as it would be necessary for a swept wing,

$$\overline{\boldsymbol{q}} = [\bar{u}(x,y), \bar{v}(x,y), 0, \bar{p}(x,y)]^T. \tag{2}$$

The small-amplitude ($\varepsilon \ll 1$) perturbations $\widetilde{\boldsymbol{q}}$ are composed of amplitude functions, which are taken to be two-dimensional functions of the resolved coordinates x and y (all three velocity components are essentially considered in the analysis), while the homogeneity of the linearized Navier-Stokes solved along the spanwise direction, z, and time, t, permits introducing eigenmodes along both of these coordinates,

$$\widetilde{\boldsymbol{q}} = \widehat{\boldsymbol{q}}\; e^{i\{\beta z - \omega t\}}, \tag{3}$$

with $\widehat{\boldsymbol{q}} = [\hat{u}(x,y), \hat{v}(x,y), \hat{w}(x,y), \hat{p}(x,y)]^T$. The spanwise wavenumber, β, is related with the periodicity length along the homogeneous spanwise direction, L_z , through $\beta = {2\pi}/{L_z}$. In the temporal framework considered here, ω is the complex eigenvalue, to be determined as part of the analysis. Its real part is related with the frequency of the eigenmode

($\omega_r = 0$ denoting a stationary and $\omega_r \neq 0$ a travelling disturbance), while a positive imaginary part, $\omega_i > 0$ corresponds to exponentially amplifying disturbances. Since both $\boldsymbol{q}$ and $\overline{\boldsymbol{q}}$ are real, while $\widehat{\boldsymbol{q}}$ and ω are complex in general, complex conjugation is introduced in equation (1). A partial-differential-equation-based eigenvalue problem of the form

$$-i\omega\, \boldsymbol{B}\ \widehat{\boldsymbol{q}}\ = \boldsymbol{A}(\overline{\boldsymbol{q}}; Re, \beta)\ \widehat{\boldsymbol{q}} \tag{4}$$

is then obtained for the complex eigenvalue ω. The structure of the discrete linear operators $\boldsymbol{A}$ and $\boldsymbol{B}$ on which the subsequent analyses are based can be found in [14] for the flat-plate boundary layer and in [7] for the stalled wing. The matrix $\boldsymbol{A}$ is treated as dense, is stored and operated upon using linear-algebra software on distributed memory supercomputers. The near-diagonal structure of $\boldsymbol{B}$ permits avoiding storage of this matrix. A massively parallel implementation of the Arnoldi algorithm is used in order to recover a window of the eigenspectrum which contains the physically-interesting most unstable/least stable eigenvalues. Implementation details of the parallel solution algorithm may be found in [9].

2.2 The basic flows considered

Laminar separation bubble on a flat-plate

A non-similar inverse formulation of the boundary-layer equations on a flat plate was used to obtain the basic flow. Two-dimensional direct numerical simulations of the Navier-Stokes equations were dismissed in this context, in order to avoid unsteadiness of the solution related with instability mechanisms. The displacement thickness δ^* distribution with the streamwise coordinate x has been imposed, such that a separation bubble exists, with a peak reversed-flow velocity ~1% of the far-field. The streamwise and wall-normal extension of the basic flow is chosen to be $L_x \times L_y = [152, 370] \times [0,40]$, lengths being scaled with the boundary layer displacement thickness at the inflow boundary, δ^*. At these parameters the inflow and outflow Reynolds numbers, respectively, are 450 and 700, such that amplification (in the local, spatial sense) of Tollmien-Schlichting instability is expected within the domain analyzed, while the global mode is stable. The basic flow velocity component, $\bar{u}(x, y)$, is shown on the left part of Figure 1.

The NACA0015 airfoil at stalled conditions

The two-dimensional steady flow around an airfoil at angle of attack α = 18° at a chord-length based Reynolds number $Re_c = 200$ is used as the basic flow. Conformal mappings have been used to generate analytically-defined O-type grids and a Jukowski transformation closely approximating the NACA0015 airfoil has been employed in order to obtain an exact match of the airfoil surface with the curvilinear coordinates used in the BiGlobal instability analysis. The steady laminar basic flow was obtained using the incompressible version of the unstructured finite-volume solver *CDP*, developed at the Stanford Center for Turbulence Research. Further information on the interpolation and conformal mapping processes, alongside the pertinent resolution studies, have been presented elsewhere [7]. The streamwise basic flow velocity component, $\bar{u}(x, y)$, is shown on the right part of Figure 1.

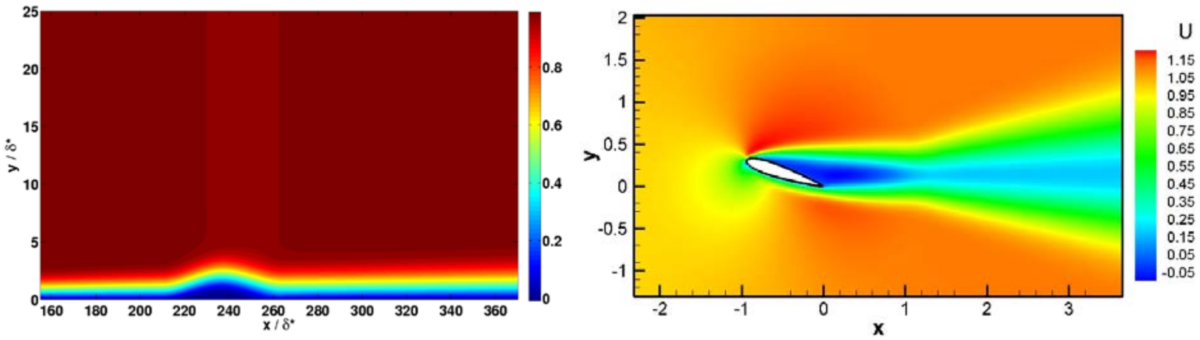

Figure 1 Streamwise velocity components of the two-dimensional basic flows corresponding to a laminar separation bubble on a flat-plate at subcritical conditions for global mode instability (*left*) and a Jukowski airfoil at angle of attack of 18° at conditions favoring amplification of the global mode of the massively separated flow (*right*).

2.3 Some concepts from critical-point theory

The organized structures in a flowfield can be characterized through the identification of the physical locations where the velocity vanishes (i.e. critical points), the behavior of the streamlines in the vicinity of these points, and the manner in which the critical points are connected by the dividing streamlines. An exhaustive description of the theory of critical points may be found in several sources, eg. [4, 6, 8, 3]; for completeness, the main aspects concerning incompressible flows are summarized next.

For a general three-dimensional flow field, having a velocity vector $\boldsymbol{v} = (u, v, w)^T$ defined on the Cartesian coordinate system $Oxyz$, critical points are defined as the spatial locations where all components of the velocity vanish and the slope of the streamlines is undetermined. The flow field can be locally expanded around the critical points using Taylor series. Considering only the linear terms in the expansion, the local behavior of the streamlines is determined by the Jacobian matrix, $\boldsymbol{J}$:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \boldsymbol{J} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ or, explicitly, } \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \tag{5}$$

where the entries of the Jacobian matrix, j_{ij}, are real constants and the critical point has been set as the origin of the coordinate system. In the case of free-slip points, i.e. critical points in the absence of a solid wall, j_{ij} are the elements of the rate-of-deformation tensor evaluated at the origin. A second type of critical points (denominated no-slip points) exists, in which the slope of the streamlines is undetermined on a solid wall. Equation (5) can still be used for characterizing the properties of no-slip critical points, introducing $(\dot{\ }) = d(\ \ )/\,d\tau$, with τ a transformed time-variable, defined by $d\tau = y\, dt$. The elements of the Jacobian matrix no longer correspond to the rate-of-stress tensor, but second derivatives of its elements in y. The limit $y \to 0$ defines the surface streamlines or lines of surface shear stress, which will be the main object of the present analysis.

In the most general case there are three planes containing solution trajectories originating at a critical point. These planes are defined as the eigenvector planes of the Jacobian matrix. Let λ be the eigenvalues of the Jacobian, satisfying the characteristic equation

$$\lambda^3 + P\,\lambda^2 + Q\,\lambda + R = 0, \tag{6}$$

where $P,\ Q$ and R are the scalar invariants of the Jacobian matrix. The invariant P is the divergence of the velocity field, which is identically equal to zero due to incompressibility. The cubic characteristic equation (6) can have (i) three different real roots, (ii) all real roots with at least two of them equal, (iii) and one real- and a pair of complex roots. Real- and complex solutions are separated on the $QR-$ space by the curve

$$S_1 = +27\,R^2 + 4\,Q^3 = 0. \tag{7}$$

Complex eigenvalues correspond to swirling flow regions, and are associated with $S_1 > 0$ while real eigenvalues correspond to $S_1 < 0$. The structures of two different flow fields are said to be topologically equivalent if the classification and connections of the critical points are identical, even though the velocity magnitudes or critical point locations are different. Nevertheless, bifurcations in the topology description can appear under small changes in the flow field. The borderline topology between two different topology descriptions is said to be structurally unstable. This can be due either to critical points where at least one eigenvalue is $\lambda = 0$ (a situation defined as *local degeneracy*) or saddle-to-saddle connections (*global degeneracy*).

What is significant in the present context is that a random small-amplitude perturbation of the structurally unstable flow field is sufficient to alter the description of the topology. Given that two-dimensional flows are structurally unstable, the objective of the present analyses then becomes the identification of flow patterns resulting from linear superposition of small-amplitude eigenmodes upon the basic flows described earlier, with particular focus being placed on the stalled-wing geometry, where the basic state has been found to be unstable to the global eigenmode of the massively separated flow [7].

3. Results

3.1 Eigenspectrum computation on the flat plate

Analyses on the flat plate were performed at discrete values of the spanwise wavelength parameter and associated spanwise periodicity length in the ranges, $\beta \in [0, 3)$ and $L_z \in [2, \infty)$, respectively. The stationary global mode was found to be least stable at a spanwise wavenumber $\beta \approx 0.15$. A resolution of 360×65 collocation points along the streamwise and wall-normal direction, respectively, was required for the convergence of the most interesting part of the eigenspectrum. The associated computing requirements were 144 Gbytes of distributed memory and 2 hours of wall-clock time on 144 processors of the Mare Nostrum computing facility. The three-dimensional domain considered for the reconstruction and topological analyses of the flow is $L_x \times L_y \times L_z = [152, 370] \times [0,40] \times [0,4\pi/\beta]$, containing the entire computational domain along the streamwise and wall-normal directions and two complete spanwise periods along the homogeneous spanwise direction.

3.2 Eigenspectrum computation on the NACA0015 airfoil at an angle of attack

Instability analyses in this flow were performed at a chord-length Reynolds number $Re_c = 200$, at which the two-dimensional basic flow at an angle of attack $\alpha = 18^{\circ}$ is steady and laminar. This permits identification of the distinct instability mechanisms at play in massively separated flow at their birth, prior to them leading flow to unsteadiness and obscuring the physical interpretation of the results. At these conditions, flow separates downstream of the leading edge, immediately after the leading-edge suction peak. The convergence of the eigenspectrum window containing the most unstable eigenvalues required the use of 249×250 collocation points. Owing to the dense linear algebra operations used (numerical details of this approach are discussed fully in [9]), the discretized matrix requires $O(1Tb)$ of distributed memory for its storage, while computation of a Krylov subspace of dimension $m_{Kryl} \approx 1000$ takes $O(24hrs)$ of wall-time on 1024 processors of the Mare Nostrum computing facility; the same computation on 2048 processors of the Blue Gene/P facility at the Forschungszentrum Jülich requires $O(12hrs)$ of wall-time for each wavenumber parameter value, β. The computational domain used extends 16 chords in the wall-normal direction, although convergence of the eigenvalue corresponding to the leading stationary eigenmode is obtained with a shorter domain extension of 11 chords in the wall-normal direction. The eigenspectrum corresponding to $\beta = 1$ is shown in figure 2, where it is compared with that at $\beta = 0.15$ on the flat plate. While the global eigenmode is stable on the flat-plate, its counterpart on the wing is unstable, suggesting spanwise periodic modification of the two-dimensional flow analyzed on account of self-excitation of the global mode of separation.

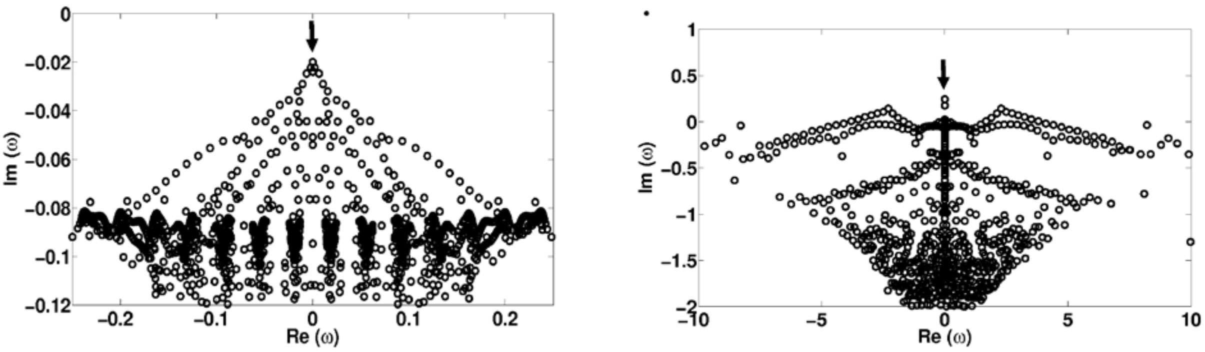

Figure 2 Global eigenvalue spectra corresponding to the separation bubble in a flat-plate boundary layer at $\beta = 0.15$ (*left*) and that on the NACA0015 airfoil at $\beta = 1$ (*Right*). The leading global modes used in the respective three-dimensional flow reconstructions are highlighted by arrows.

Theoretical considerations linking the small-amplitude global eigenmode with the generation and evolution of critical points within the three-dimensional domains considered are presented in [10] for the flat plate and in [11] for the NACA0015 airfoil. Here, attention is paid on comparison of the respective results, focusing on two phenomena explained by application of critical point theory to global linear instability analysis results: (a) *field topologies* resulting from the global eigenmode in the flat plate, in particular *U-separation* [6], (b) *wall-surface topology* (wall-streamlines) comparisons between results on the flat-plate and the NACA 0015 airfoil, and (c) the birth of *stall cells* on the airfoil.

However, prior to presenting results of the topological analysis of the composite flowfields resulting from amplification of the global mode, the issue of comparison between the present results and the classic linear theory predictions of Tollmien-Schlichting instability is addressed on the flat-plate, where the selection of flow parameters is consistent with linear amplification of TS instability.

In a local analysis context, Tollmien-Schlichting waves are recovered as solutions of the Orr-Sommerfeld equation around a basic state which varies as one moves downstream along the separation bubble and the leading eigenmode at each spatial location (the TS-wave) is related to that at subsequent positions. In the present global approach, the wave-like disturbances are recovered as a family of eigenmodes, discretized according to the parameters (i.e. domain extension and resolution) in which the EVP is solved. Each discrete eigenmode represents the evolution of a wave with fixed frequency, a result shown on the left part of figure 3. The subsequent development of a wave-packet may be reconstructed in a global context by the non-modal behaviour of the superposition of several eigenmodes such as that shown in the left part of figure 3. Shown in the right part of the same figure are N-factor curves computed from results of the local and BiGlobal approach; they are found to be in rather good agreement within the zone of strong amplification, inside the laminar separation bubble.

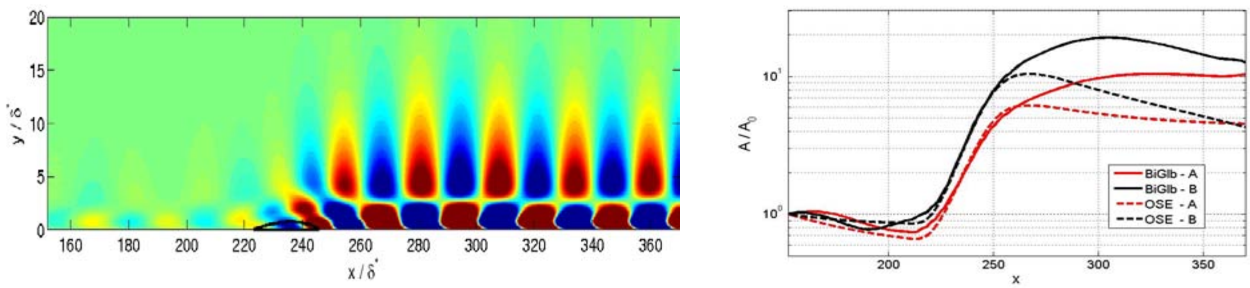

Figure 3. *Left*: Amplitude function corresponding to an oblique Tollmien-Schlichting wave excited by the laminar separation bubble (the outline of which is also shown). *Right*: Comparison of the disturbance amplitudes of two fixed-frequency wave-like disturbances recovered from global and local analyses.

3.3 Highlights of the flow topology on the flat-plate: U-separation

Turing to the global mode and the three-dimensional flowfield reconstruction, the loci of the critical points keep moving on the QR plane, in line with the continuous increase of the amplitude of the eigenmode used in the linear reconstruction. Figure 4 shows two characteristic situations, one at low amplitude of the global mode superimposed upon the basic state and one at which the amplitude at which this mode finds itself in the laminar separated boundary layer flow is higher, but still linear. Shown are wall- and field streamline trajectories, alongside the location and nature of the critical points analyzed [10]. As can be identified by the larger-amplitude image (and all other reconstructions at yet higher amplitudes, not shown here) the effect of the global mode on the initially two-dimensional flow is the appearance of the U-separation flow pattern, first defined in early classifications of flow separation [6, 8, 3].

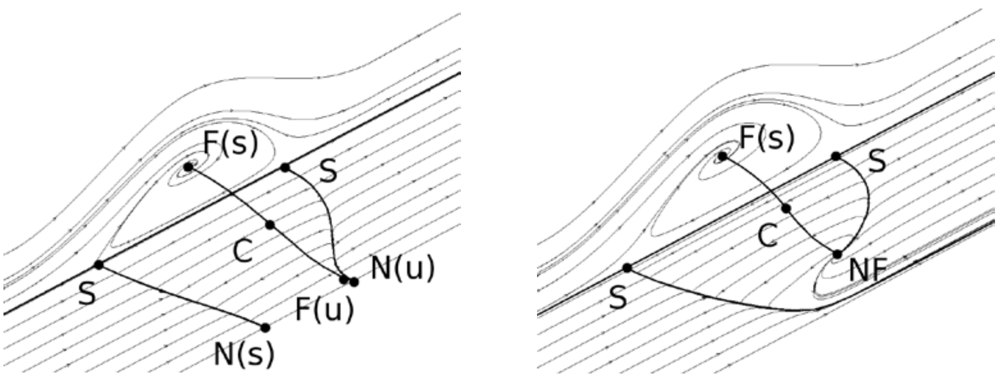

Figure 4 U-separation [6, 8, 3] produced by the presence of the leading global eigenmode of laminar separation bubble on a flat plate at small amplitude (*left*) and slightly larger – but still linear (*right*). Superposed to the field- and wall-streamlines are the locations of the critical points, as well as their nature: S: saddle; C: center; N: node; F: focus; NF: Node-focus; (s): stable; (u): unstable.

3.4 Comparison of surface streamline topologies on the two separated flow configurations

The disturbance components of the leading three-dimensional stationary global mode on the flat-plate and the stalled airfoil present analogous characteristics in terms of the spatial distribution of the respective amplitude functions, first seen in [13] on a model flow. The streamwise velocity component $\hat{u}$ is centered around the primary separated flow region, and attains its peak value near the primary reattachment location. A spanwise velocity component $\widehat{w}$ with amplitude of the same order of magnitude as $\hat{u}$ is also part of the global eigenmode recovered. This amplitude function attains its minimum and maximum values at the spanwise locations where the streamwise component vanishes, resulting into a fully three-dimensional eigenmode. In the case of the flat-plate, the spanwise velocity component vanishes on a streamwise location between the separation and reattachment lines of the basic flow; However, in the case of the airfoil, this component vanishes at an additional line, which overlaps to the reattachment line of the basic flow [11]; this additional line arises on the airfoil on account of the satisfaction of the Kutta condition.

In the full three-dimensional field reconstructions that follow in both the flate plate and the airfoil, the amplitude functions of the respective global eigenmodes are normalized such that the maximum velocity component is equal to unity; the global modes of each flow are then multiplied by a linearly small amplitude and are superposed to the respective basic state. A new set of no-slip critical points emerges at the surface, at the locations where the wall-shear components of the reconstructed flow vanish. Focusing on wall-streamline topologies only, one notes that with increasing perturbation amplitudes, the topologies of the surface streamlines eventually undergo a series of bifurcations – one for the plat-plate and three for the airfoil – leading to the topological patterns shown in figure 5. The range of amplitudes in which these bifurcations occur are $O(10^{-2})$ in the flat-plate case and $O(10^{-3})$ in the airfoil case, remaining in the linear regime. All bifurcations experienced by the two flows, starting from two-dimensionality and progressively resulting in the states described herein and in [10, 11], are completed within the linear regime, and no new bifurcations are experienced by either flow, even if the amplitudes of the linear global modes are (artificially) increased, even up to $O(1)$, where the linearity approximation ceases to be valid.

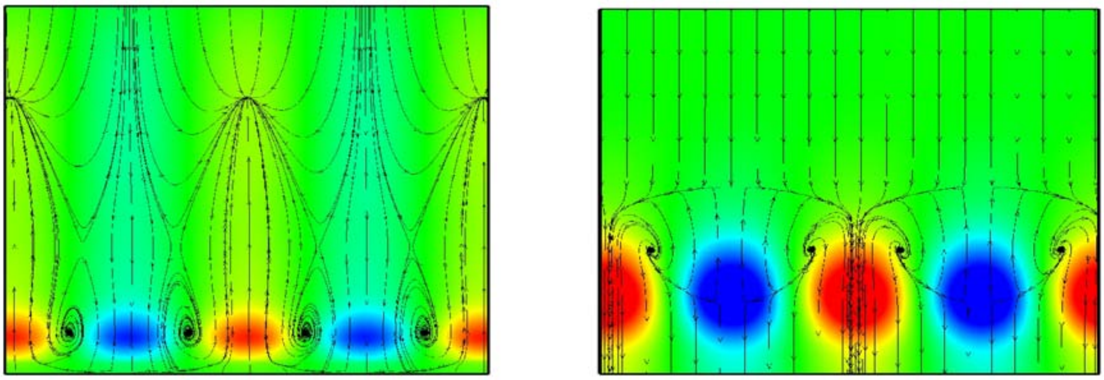

Figure 5 Surface streamlines resulting from the linear superposition to the two dimensional basic flows of their leading three-dimensional global mode. The contours are streamwise/chordwise component of the wall-shear. Shown is the streamwise domain extent around the primary laminar separation. *Left*: Flat-plate. *Right*: Airfoil.

3.5 The birth of stall-cells on the airfoil

The final surface streamline topology of the reconstructed three-dimensional flowfields in both the flat-plate and the airfoil shows that the connex reversed flow region present in the two-dimensional basic flow has broken periodically in the spanwise direction, to give rise to independent separated regions. In these regions, the streamlines are organized into counter-rotating swirling patterns, resembling the stall cells observed experimentally. The streamlines on the flat-plate extend to the infinite downstream, showing that the semi-infinite flat-plate is a simplification of the problem of the airfoil that looses part of the physics, as no trailing edge exists. On the other hand, on the airfoil geometry, the Kutta condition imposes that the ultimate detachment of the streamlines occurs at the trailing edge; this gives rise to additional critical points and, consequently, to additional bifurcations compared with those found on the flat-plate separation, as well as to an altogether substantially more complicated topological flow description; a full discussion of the theoretical results on either geometry may be found in [10, 11]. The key result in the context of the airfoil is that the surface streamlines recovered by linear superposition of the (stationary, unstable) global mode upon the massively separated flow are identical to those of the well-known stall cells, recovered experimentally [2, 16] and described from a phenomenological point of view in the literature (e.g. [12, 5]). Such an image of a three-dimensional flowfield reconstruction for the NACA0015 wing at an angle of attack of 18 degrees may be seen in figure 6.

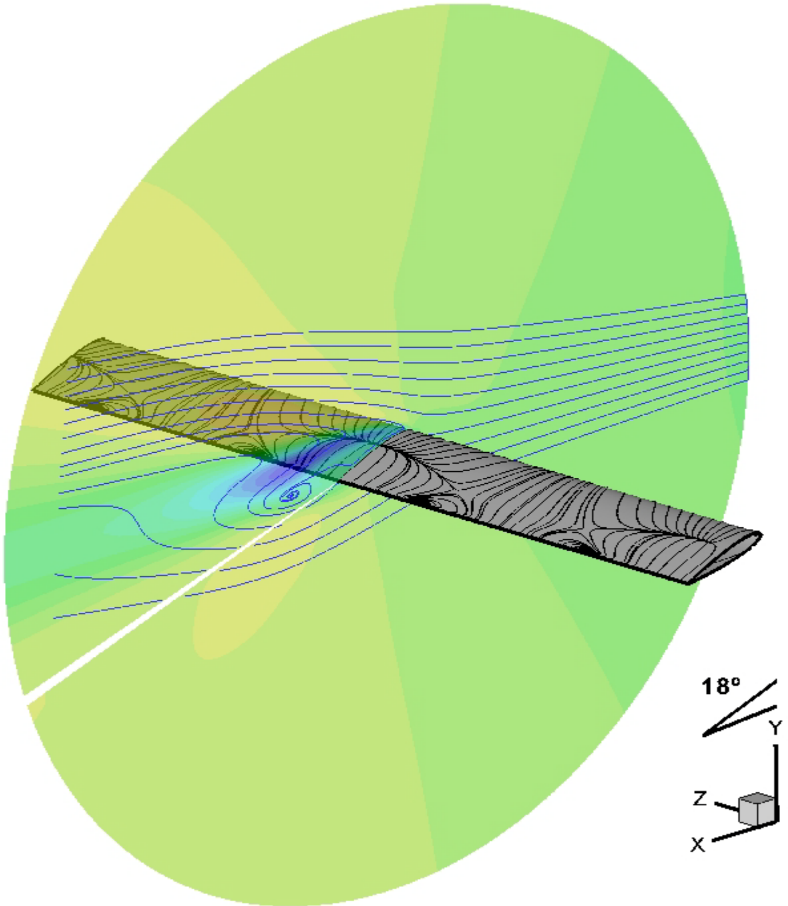

Figure 6 Three-dimensional reconstruction of the stall cells generated by the linear superposition to the two-dimensional basic flow around a stalled airfoil of its leading stationary three-dimensional eigenmode. Graphically superposed are streamlines on the plane $\beta z = 0$, as well as the color-coded amplitude function of the streamwise velocity component.

Discussion

Two separated flow configurations of both academic and industrial interest have been analyzed using a combination of BiGlobal linear- [13] and critical point [4, 6, 8, 3] theories: laminar-separation bubble in boundary-layer flow on a flat plate at high-Reynolds number, at conditions where both TS- and global-mode instabilities are present, as well as massively separated flow on a high-angle of attack NACA0015 wing at low Reynolds numbers, at which the global mode instability dominates. While in the first configuration the linearly unstable shear layer might obscure not only the linear global instability but also its relation with the topological changes on the plate/wing surface, in the latter flow configuration the fact that the global mode dominates all other types of instability permits putting the interpretation of results on firm theoretical ground.

It is known from critical point theory [4] that any steady two-dimensional flow, including the laminar separation bubble boundary layer and the massively separated flow on the NACA0015 wing of interest here, is structurally unstable due to the homogeneity assumption in the third spatial direction and the resulting singular Jacobian matrix. As a consequence, any three-dimensional disturbance will give rise to a three-dimensional flow field, regardless of the amplitude of such disturbance. Here, the structural changes that these basic flows experience on account of the two predominant linear instability mechanisms that they support, namely incoming amplified Tollmien-Schlichting waves and self-excited global modes [14], have been studied in detail.

After making a brief, satisfactory comparison between the results delivered by the local and the global approach on the local spatial amplification of the Tollmien-Schlichting instability, critical point theory has been employed in both the flat plate and the stalled wing in order to study the topology of field- and wall-streamlines generated by linear superposition of the leading global mode upon the respective basic state.

The first finding of significance has been the identification of the U-separation phenomenon, one of the multitude of separated flow patterns first identified by Hornung and Perry [6] and subsequently discussed in detail by Chong *et al.* [3], as being the result of amplification of the global mode of separation. The second key finding of the present analysis has resulted from examining surface streamline topologies also from the point of view of critical point theory. Here, it has been shown [9, 10] that both on the flat plate and the stalled wing, at disturbance amplitudes small enough for the linearity assumption to be valid, the amplification of the global mode leads to a spanwise breakdown of the separated region with a fixed periodicity length, described by a set of connected no-slip (node and saddle) critical points on the wall surface. The effect of the three-dimensional stationary amplified global mode is to modulate the extension in the basic state of the connex region of reversed flow, eventually breaking it to periodic cells having a spanwise periodicity length predicted by global linear instability theory. This spanwise periodic breakdown gives rise to independent separated flow regions, in which the structure of the streamlines is organized around two counter-rotating foci. In addition, counter-rotating structures appear in the surface streamlines, bearing strong resemblance to the stall cells (also referred to as "owl-face structures" or "mushroom structures") observed in experiments on airfoils close to and beyond stall [2, 16, 12]. Since the origins of both phenomena of U-separation and stall cells has been identified in the same linear global mechanism, presently theoretically-founded flow control mechanisms are being explored in order to control either instability.

The present quantitative description of the flow field topology, as one of a steady laminar two-dimensional flow perturbed by its leading global mode, puts for the first time on firm theoretical basis the multitude of phenomenological descriptions of the stall-cells observed in experiments on airfoils close to and beyond stall [16, 2, 12] (no attempt to describe theoretically U-separation is known to the authors). Moreover, it is worth adding here that the global mode of separation predicted by [14] has since been found to have amplitude functions of analogous structures on geometrically different but topologically analogous configurations, such as flow over the NACA0012 wing at an angle of attack [15] and low-pressure turbine blades [1]. On account of the Kutta condition present in both latter configurations, it is expected that the stall-cell mechanism identified on the NACA0015 airfoil will also be operative on generic airfoil and low-pressure turbine blade configurations. Such a speculation needs to be examined by analyses analogous to that presented herein and in [9, 10]. The effect of crossflow, as encountered on account of sweep in most wings of industrial interest is another interesting line of future investigations. Finally, as mentioned several times here, the topological bifurcations described occur at disturbance amplitudes of maximally $O(10^{-2})$, levels at which the linearization hypothesis is still valid. Since the leading global eigenmode is unstable at the conditions monitored, the perturbation amplitude will grow exponentially and the flow field will undergo all the topology bifurcations described herein, until non-linear effects appear and saturate further growth. Further work employing three-dimensional direct numerical simulations is currently underway, in order to quantify the precise role of nonlinearity in the scenarios discovered herein.

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